# REDUCED FORM PE-FUND ASSET MODELING

## CEQURA Conference 2018 Talk

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# AGENDA

Agenda

1. Introduction
2. Probability Space
3. Structural vs. Reduced Form Approach (in the PE Context)
4. Exit Dynamics Regression
5. Data & Model Estimation
6. Monte Carlo Simulation
7. Conclusion

# 1 INTRODUCTION

# 1.1 WORKING PAPER

| | |
|---|---|
| **Title** | Reduced form private equity fund asset modeling |
| **Status** | Working paper |
| **Co-authors** | Axel Buchner, Georg Schlüchtermann |
| **Idea** | Use reduced form credit risk approach to model the exit behavior of private equity fund investments |
| **Aim** | Quantify the relationship between exit timing and exit performance on asset level |
| **Application** | Monte Carlo simulation tool to estimate ”Cash-Flow-at-Risks” of private equity portfolios |

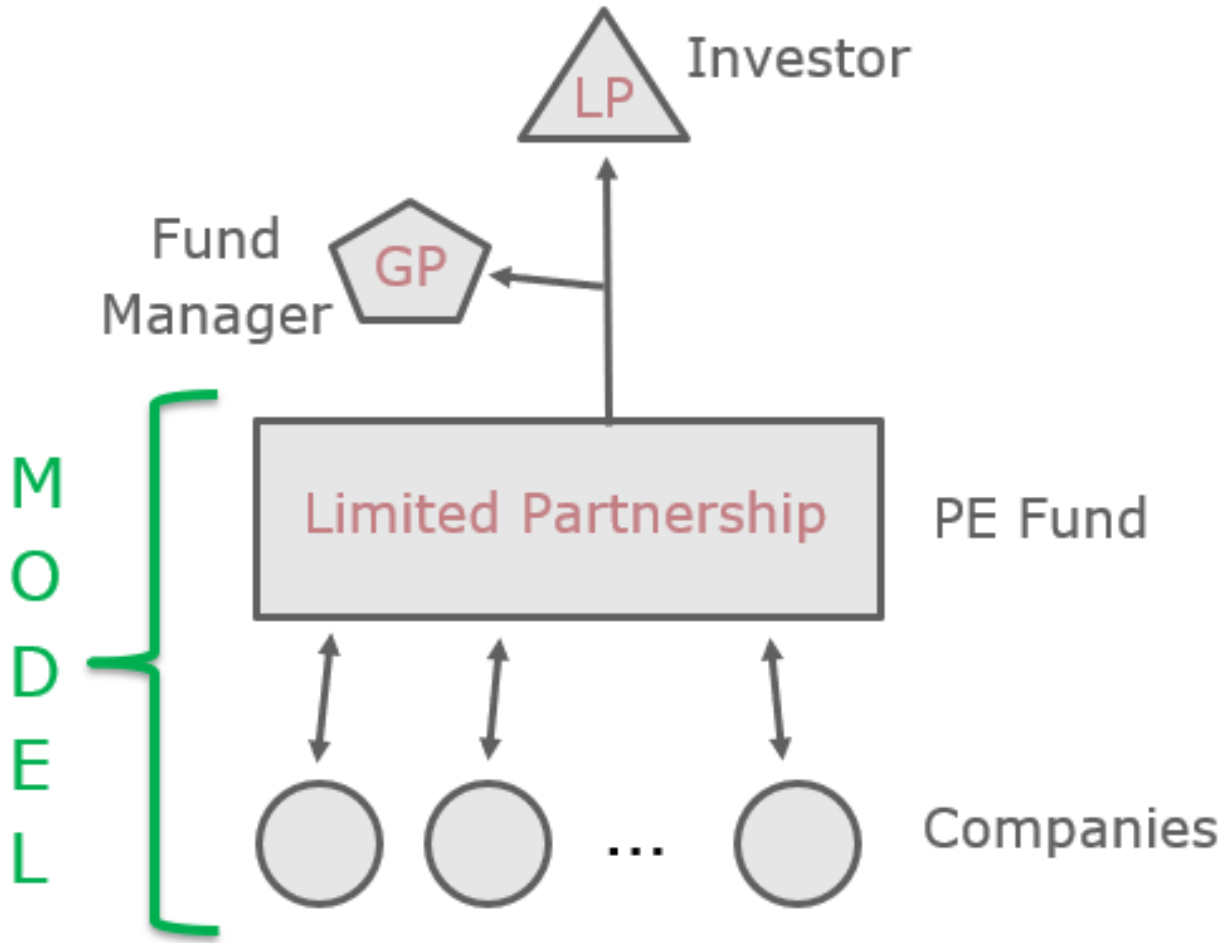
# 1.2 PRACTICAL EXAMPLE

Assume:

- You hold a PE fund consisting of 3 (or 15) underlying companies.
- The fund manager values your share to 1 million €.
- No secondary market for private equity exists.
- You have to wait until the fund manager sells the companies.
- Fund managers charge no fees.

**How much money will you receive? When?**

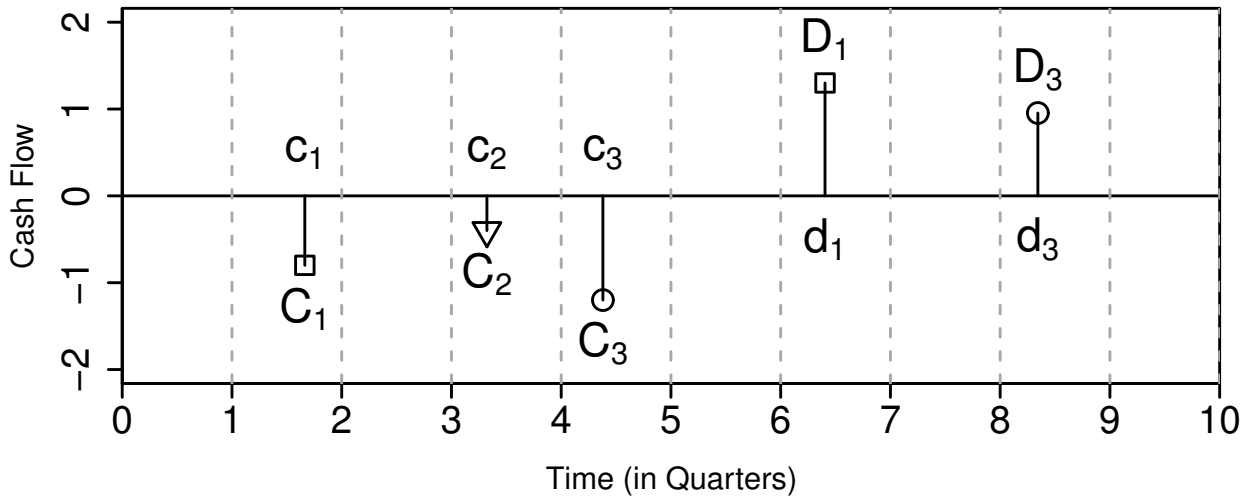
# 1.3 PRIVATE EQUITY FUND STRUCTURE



# 2 PROBABILITY SPACE

All fund cash flows are modeled by a marked point process (MPP)

$$(\mathcal{C}_i, \mathcal{D}_i) = (c_i, C_i, d_i, D_i)$$




| Event Type | Timing | Amount. |
|---|---|---|
| Investment | $c_i$ | $C_i$. |
| Divestment | $d_i$ | $D_i$. |

Let $N^{(d)}(t)$ count all exit events happening before time t.

$$N^{(d)}(t) = \sum_{i=1}^{n} 1_{\{d_i \leq t\}}$$

In addition there exist 4 auxiliary processes:

**Value process** $V_i(t) = \mathring{V}_i(t) + \upsilon_{t,i}$ (proxy for true asset value $\mathring{V}$)

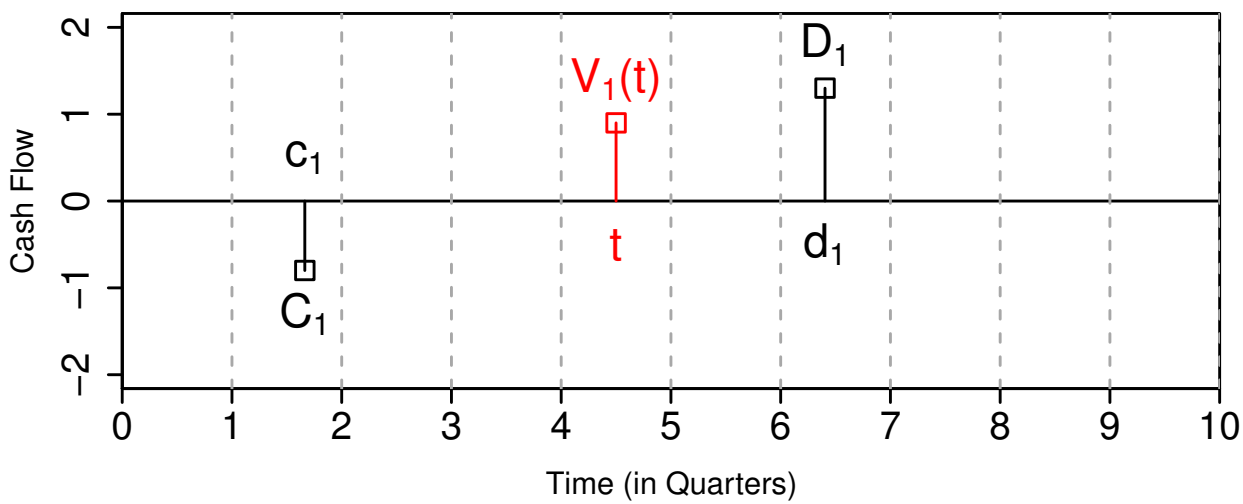
**Covariate process** $X_i$ (public market factors, company details)

**Censoring process** $Z_i^{(\text{censoring})}$ (indicates active investment)

**Filtering process** $Z^{(\text{filtering})}$ (due to quarterly reporting in PE)

# 3 STRUCTURAL VS REDUCED FORM

**Definition 1:** A private equity fund (PEF) model on asset level connects the intermediate proxy valuation process $V_i(t)$ with the resulting final divestment cash flow $\mathcal{D}_i$ for $t \in (c_i, d_i)$.




**Remark** The investment part $\mathcal{C}_i$ is ignored in our exit model.

**Definition 2:** A structural PEF model on asset level is defined in terms of the observable and unobservable firm value processes V and $\mathring{V}$

$$D = C + \int_c^d dV(s) = C + \int_c^d d\mathring{V}(s)$$

with

$$d = \inf\{t > c : V(t) \in E(t)\} \wedge \inf\left\{t > c : \mathring{V}(t) \leq 0\right\}$$

where E(t) is the dynamic exit acceptance set.

**Most authors use structural approaches to model PE assets, but ...**

- Structural models require detailed information.
- Incomplete information settings are inherent to private equity.
- Scarce data: quarterly reporting, GP appraisals, stale pricing ...

**Definition 3:** A reduced form PEF model on asset level is defined in terms of the observable proxy value process V and a stochastic multiplier $m_D$ that is modeled conditional on exit timing, i.e.

$$D = m_D(t|d) \cdot V(t)$$

with

$$d = \inf\left\{t > c : \; N^{(d)}(t) > 0\right\}$$

- Reduced form = model based on observable quantities.
- The exit timing d is determined by the exit counting process $\mathbf{N}^{(d)}$.
- V(t) is not perceived as stochastic process, but realized data.

# 4 EXIT DYNAMICS REGRESSION

# 4.1 EXIT DYNAMICS REGRESSION DESIGN

Marginal parametric models for the bivariate dependent variable:

$$\textbf{Timing} \quad y = d - t = m_y(\mathbf{X}|\xi_y) + e_y$$
$$\textbf{Multiple} \quad Y = \frac{D}{V(t)} = m_Y(\mathbf{X}|\xi_Y, y) + e_Y$$

where $\xi_Y$, $\xi_y$ are the model parameters
and $e_Y$, $e_y$ are (possibly mutually dependent) error terms.

- **Multiple** is modeled conditional on **Timing**.
- Parametric models are more easy to estimate and simulate.

Intensity based approach

$$\mathbb{E}\left[\int_0^\infty dN_i^{(d)}(u)\right] = \mathbb{E}\left[\int_0^\infty h_i(u|\mathbf{X}(u))\,du\right]$$

Multiplicative hazard formulation (a.k.a. Cox Regression)

$$h_i(t|\mathbf{X}(t)) = h_0(t)\exp(\beta\mathbf{X}(t))\,Z_i^{(\text{censoring})}(t)$$

Parametric Weibull base hazard

$$H_0^{(\text{wb})}(t|\xi_y) = \int_0^t h_0^{(\text{wb})}(u|\xi_y)\,du = \left(\frac{t}{\text{scale}_{\text{wb}}}\right)^{\text{shape}_{\text{wb}}}$$

 Stepwise integration of $H^{(\text{wb})}$ due to time-varying covariates.

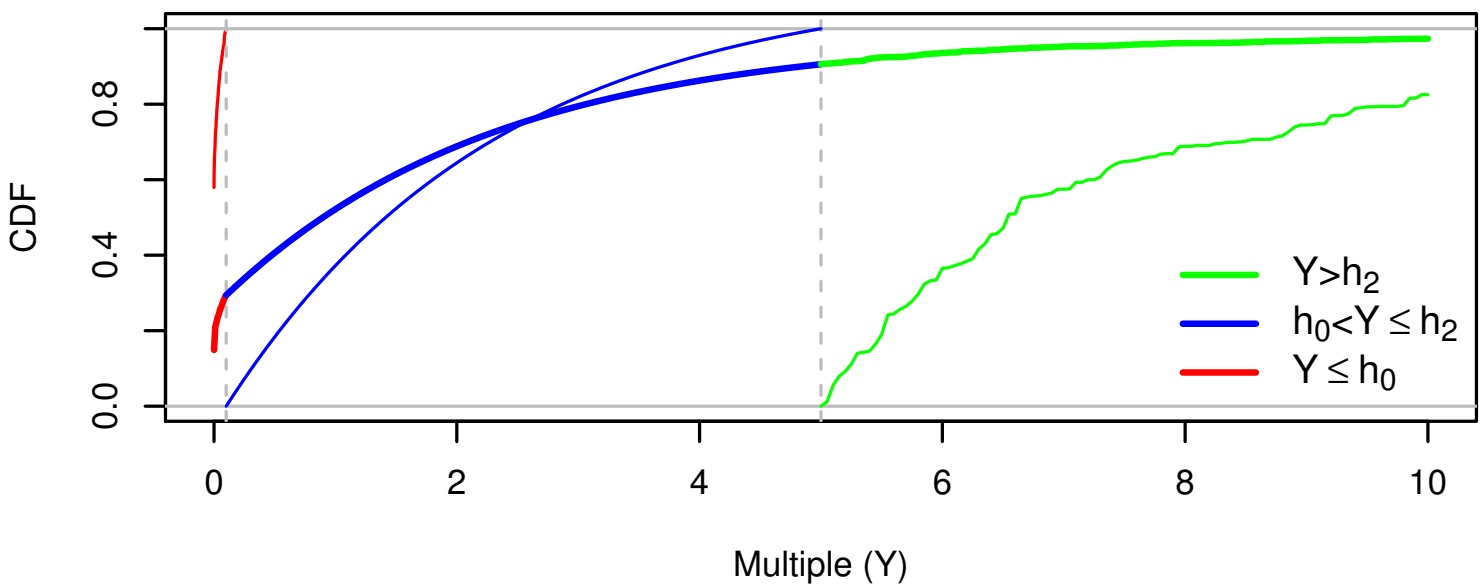
# 4.3 MULTIPLE REGRESSION

⚠ Private equity multiples are option-like, i.e. many defaults and some very high returns (25% defaults and 10% with return > 400%)

⚠ Normal and log-normal assumption not valid for $Y = \frac{D}{V(t)}$

💡 Truncation: exclude negative and positive outliers

💡 Use three part hurdle model (use empirical CDF in outlier region, Gamma distribution in-between)



# 5 DATA & MODEL ESTIMATION

# 5.1 DATA & MODEL ESTIMATION

- Asset level data set of 144 Buy Out and 98 Venture Capital funds
- Both models (Timing & Multiple) are fitted by maximizing the parametric likelihood functions
- Exit Multiple model considerations:
  - Data subset:
    - Exit event before 2017-01-01 (realized investments)
    - Entry event before 2010-01-01 (”quick-flip bias”)
    - RVPI-ratio $>$ 10% (economic relevance)
  - Resampling based estimation approach to account for auto-correlation of quarterly reported net asset values (i.e. sample one observation per company ID in each iteration)

# 5.2 EXPLANATORY VARIABLES

| | Variable name | Timing | Multiple |
|---|---|---|---|
| $x_1$ | Holding period: $t - c$ | no | yes |
| $x_2$ | Time to exit: $d - t$ | no | yes (as $x_2 + x_3$) |
| $x_3$ | Zombie stage: $\max(0, d - c - 10)$ | no | yes (as $x_2 + x_3$) |
| $x_4$ | RVPI: $\frac{V}{C}$ | no (internal) | yes |
| $x_5$ | High yield spread | yes | yes |
| $x_6$ | Public equity performance | yes | yes |
| $x_7$ | Fund age at entry (date) | yes | no |

# 5.3 Regression results (Buy out and venture capital)

Regression results seem plausible:

- PE-funds exit their companies the faster:
  - the higher the public equity return,
  - the lower the high yield spreads,
  - the older the fund age.
- High Multiples can be expected for:
  - high public equity returns,
  - low high yield spreads,
  - short (future) time-to-exit's.
- Long (past) holding periods decrease the probability of default.
- The lower the RVPI ratio ($\frac{V}{C}$), the higher the Multiple variance.

# 6 MONTE CARLO SIMULATION

# 6.1 MONTE CARLO SIMULATION EXAMPLE

Compare two Venture Capital funds with 3 and 15 company holdings:

- All companies are already held 5 years and have an RVPI (current value to cost ratio) of 1
- 5,000 simulation iterations
- Fees are not considered
- Public market scenarios are generated by historical simulation

**Which fund is riskier?**
**How much money can you expect after 5 or 10 years (from now)?**

# 6.2 SIMULATION OUTCOME: CASH-FLOW-AT-RISK CHART

# 6.3 SIMULATION OUTCOME: EXAMPLE QUANTILES

| Cash-Flow-at-Risk | 3-company | 15-company |
|---|---|---|
| **"10% worst case after 5 years"** | | |
| 5-year horizon, 10% quantile | 0.00 | 0.34 |
| **"10% best case after 10 years"** | | |
| 10-year horizon, 90% quantile | 3.01 | 2.69 |

**Cash-Flow-at-Risk:** Maximum amount of cumulative divestment cash flow you can you expect after a given horizon with a given probability (in relation to the current valuation).

# 7 CONCLUSION

# 7.1 CONCLUSION

Our paper:

1. contributes a **marked point process** framework to describe the cash flow dynamics of private equity funds on single asset level.
2. studies the **divestment behavior** of private equity funds in a reduced form approach, where the exit multiple is modeled conditional on exit timing.
3. proposes tailored **parametric regression** models, which can include covariates that are infeasible in fund level approaches.
4. highlights the superiority of **reduced form** formulations in incomplete information settings like private equity.

**General Aim:** Model for $(d, D)$ given $V(t)$ and $(c, C)$.

| | Structural | Reduced-form |
|---|---|---|
| Stochastic Processes: | | |
| - observable | $V(t)$ | $N^{(d)}(t)$ |
| - unobservable | $\mathring{V}(t)$, $E(t)$ | - |
| Simulation Procedure: | | |
| 1. Step | **Multiple** paths for $V(t)$ and $\mathring{V}(t)$ | **Timing** d directly from survival function |
| 2. Step | **Timing** check when $E(t)$ is met | **Multiple** $m_D(t\|d) \cdot V(t)$ |

Working paper and R code are available on my blog Quant-Unit.com

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Do you have comments?